

Hydraulics

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Pressure, Force, and Area

- Pressure is defined by the equation:

$$P = \frac{F}{A}$$

P = pressure

F = force

A = area

- The SI units for pressure are Pascals

$$1Pa = 1 \frac{N}{m^2}$$

Pressure, Force, and Area (2)

- Atmospheric Pressure is $101.35kPa \approx 10^5 Pa$
- **Force vs. Pressure:**
 - Force is given by Newton's 2nd Law: $F = ma$
 - Force tells you how an object will accelerate
 - Pressure tells you how an object will feel

Pressure vs. Force



Bed of Nails Demonstration

- Force is distributed over many nails, so the pressure is reduced

Example 1

- Shaquille O'Neal weighs 150kg and wears size 22 shoes, which have an area of approximately $1,000\text{cm}^2$ each. A female weighing 60kg wears stilettos with an area of 1cm^2 on each heel and 24cm^2 on each toe. Calculate the pressure exerted by each person on the ground. Who would you rather have step on your foot?

Example 1 (2)

Force of Gravity:

$$F_g = mg$$

$$F_g = (150kg) \times \left(\frac{9.8N}{kg} \right)$$

$$F_g = 1,470N$$

Area of Shoe:

$$A = 1,000cm^2 \times \left(\frac{1m}{100cm} \right)^2$$

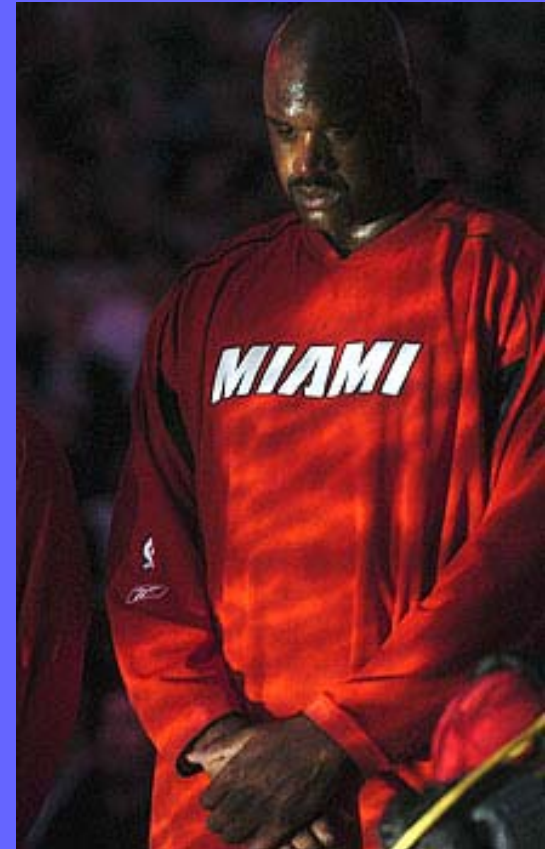
$$A = 0.1m^2$$

Pressure: (Don't forget there are 2 Shoes!)

$$P = \frac{F}{A}$$

$$P = \frac{1,470N}{2 \times 0.1m^2}$$

$$P = 7,350Pa$$



Example 1 (3)

Force of Gravity:

$$F_g = mg$$

$$F_g = (60kg) \times \left(\frac{9.8N}{kg} \right)$$

$$F_g = 588N$$

Area of Shoe:

$$A = (1cm^2 + 24cm^2) \times \left(\frac{1m}{100cm} \right)^2$$

$$A = 0.0025m^2$$

Pressure: (Don't forget there are 2 Shoes!)

$$P = \frac{F}{A}$$

$$P = \frac{588N}{2 \times 0.0025m^2}$$

$$P = 117,600Pa$$



Example 1 (4)

- So who would you rather have step on your foot?

Shaq : $P = 7,350\text{Pa}$

Woman : $P = 117,600\text{Pa}$

- Note: A large force can create only a small pressure if it is spread out over a wide area. A small force can create a big pressure if the area is tiny.

Example 1 (5)

- The same principles of pressure, force, and area are used for applications such as snowshoes, and as you will see, hydraulics.

Pressure in Fluids

- Now what about pressure in fluids?
- For gases, we know the Ideal Gas Law:
 $PV=nRT$
- So we can increase the pressure in a gas by doing one of three things:
 1. Increasing the temperature (T)
 2. Increasing the amount of gas in the system (n)
 3. Decreasing the volume (V)

1st Demonstration

- The purpose of this demo is to illustrate the effects of increasing the pressure of a gas (by increasing the number of particles) to lift a person sitting on a piece of plywood.
- Need: 7 volunteers

1st Demonstration (2)

- How much pressure was needed to lift our volunteer?

$$\text{Mass} = m_{\text{board}} + m_{\text{person}}$$

$$\text{Mass} = 2\text{kg} + 60\text{kg}$$

$$\text{Mass} = 62\text{kg}$$

$$F = (62\text{kg})(9.8\text{N/kg})$$

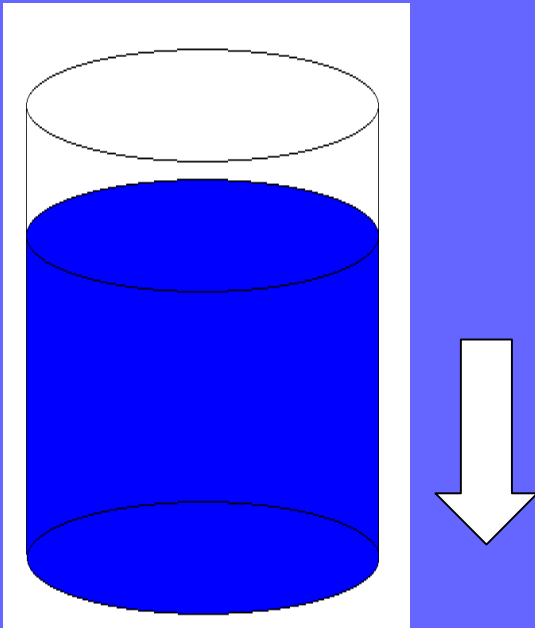
$$F = 608\text{N}$$

$$A_{\text{board}} = 0.36\text{m}^2$$

$$P = (608\text{N})/(0.36\text{m}^2)$$

$$P = 1,690\text{Pa}$$

Pressure in Liquids



Pressure Increases With Depth

- What about pressure in liquids?
- The pressure in a liquid depends on depth.
- Why?
 - Because the force on the liquid increases as the weight of the liquid above it pushes down on it.
 - The deeper you move in the liquid, the more weight pushes down.

Pressure in Liquids (2)

- Quantitatively, pressure in a liquid is related to the depth by the formula:

$$P = P_0 + \rho gh$$

- Where:
 - P_0 is the pressure of the air above the liquid (atmospheric pressure)
 - ρ is the density of the liquid ($\rho_{\text{water}} = 1,000 \text{ kg/m}^3$)
 - g is the acceleration due to gravity (9.8 N/kg)
 - h is the depth below the surface of the liquid

A Common Problem

- Diving into a swimming pool causes pressure in your ears that can be painful
- “Popping” your ears equalizes the pressure within them
- At deeper water levels one’s eardrums can burst due to the increased pressure

Pascal's Law

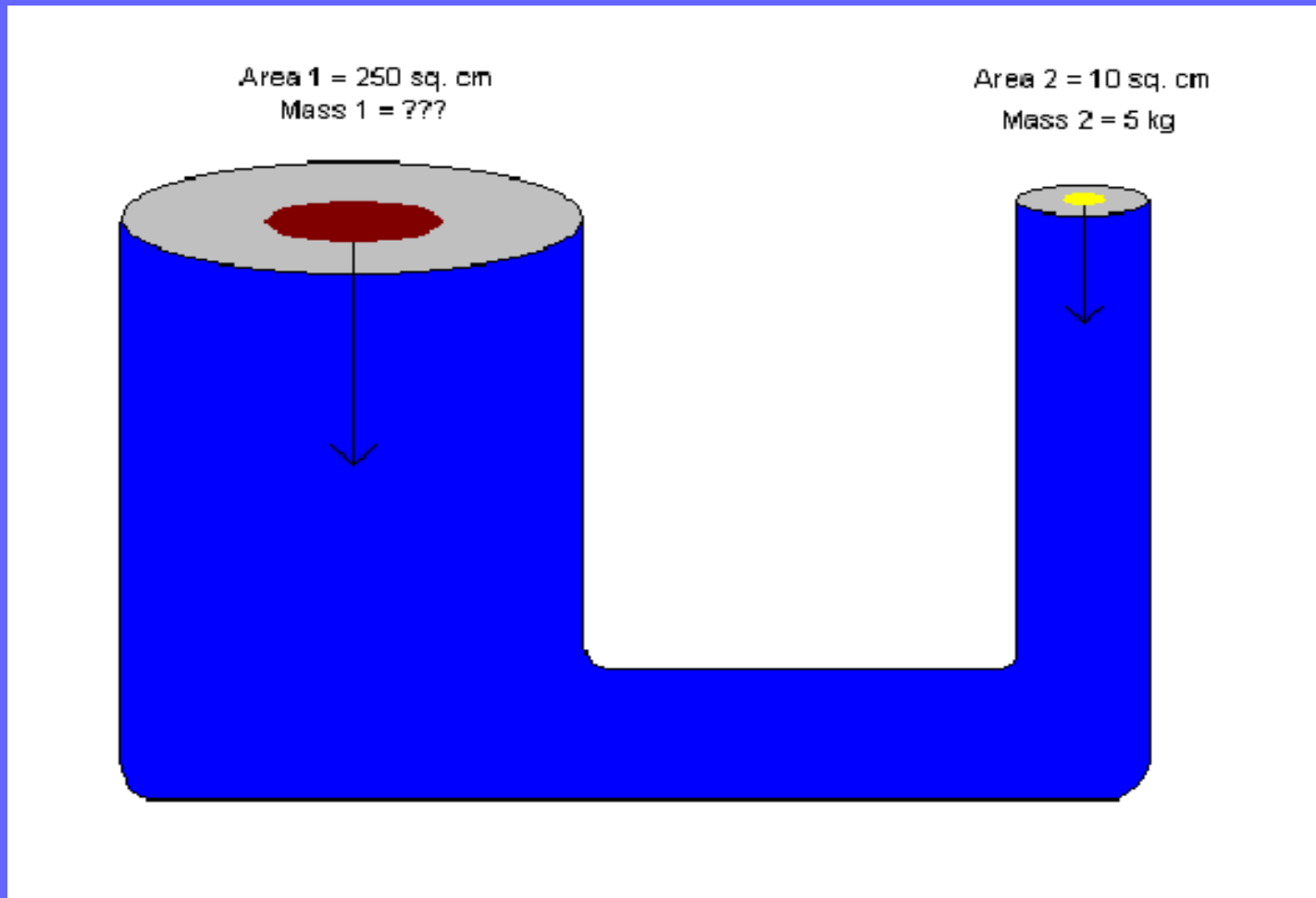
- An important law in fluid dynamics is Pascal's Law:
 - *A change in the applied pressure on a fluid is transmitted undiminished to every point of the fluid and the walls of the container.*
- From Pascal's Law we get hydraulics

Example 2

Consider the Following Situation:

Two pipes of areas $A_1=10\text{cm}^2$, and $A_2=250\text{cm}^2$, are connected to each other. The two pipes are fitted with weightless pistons that are free to slide frictionlessly inside the apparatus. The pipes are filled with an incompressible fluid, and the bottom of the pistons are directly aligned with the fluid level. If a 5kg weight is placed on the smaller piston, how much weight can be supported on the other side?

Example 2 Schematic



Example 2 (3)

- How do we approach this problem?
- Remember $P = \frac{F}{A}$, and Pascal's Law.

$$A_1 = (250\text{cm}^2) \times \left(\frac{1\text{m}}{100\text{cm}}\right)^2$$

$$A_1 = 0.025\text{m}^2$$

$$A_2 = (10\text{cm}^2) \times \left(\frac{1\text{m}}{100\text{cm}}\right)^2$$

$$A_2 = 0.001\text{m}^2$$

Example 2 (4)

$$F_2 = m_2 g$$

$$F_2 = (5\text{kg}) \left(\frac{9.8\text{N}}{\text{kg}} \right)$$

$$F_2 = 49\text{N}$$

$$P_2 = \frac{F_2}{A_2}$$

$$P_2 = \frac{49\text{N}}{0.001\text{m}^2}$$

$$P_2 = 49,000\text{Pa}$$

Example 2 (5)

- Now from Pascal's Law we know that $P_1 = P_2$, so:

$$P_1 = \left(\frac{F_1}{A_1} \right) = P_2$$

$$49,000 Pa = \left(\frac{F_1}{0.025 m^2} \right)$$

$$\Rightarrow F_1 = 1,225 N$$

$$F_1 = m_1 g$$

$$m_1 = \frac{F_1}{g}$$

$$m_1 = \frac{1,225 N}{9.8 N / kg}$$

$$\therefore m_1 = 125 kg$$

So What's The Point?

- The implication of this principle is that a smaller mass can lift a larger mass, using the “**Force-Multiplying Power**” of a hydraulic system.
- For our example, the Force-Multiplying Power is the ratio of the masses, or 25.
- But... does this really work? Can an object really lift something 25 times its own mass?

2nd Demonstration

- The purpose of this demonstration is to investigate the Force-Multiplying Power of a simple hydraulic system (reverse problem)
- The apparatus we are going to consider has the following properties:

$$A_1 = 3.6 \times 10^{-3} m^2$$

$$A_2 = 3.1 \times 10^{-2} m^2$$

$$m_1 = 3.5 kg$$

2nd Demonstration (2)

$$F_1 = m_1 g$$

$$F_1 = (3.5 \text{ kg}) \left(\frac{9.8 \text{ N}}{\text{kg}} \right)$$

$$F_1 = 34.3 \text{ N}$$

$$P_1 = \frac{F_1}{A_1}$$

$$P_1 = \frac{34.3 \text{ N}}{0.031 \text{ m}^2}$$

$$P_1 = 1,106 \text{ Pa}$$

2nd Demonstration (3)

$$P_2 = \left(\frac{F_2}{A_2} \right) = P_1$$

$$1,106 Pa = \left(\frac{F_2}{0.0036 m^2} \right)$$

$$\Rightarrow F_2 = 4N$$

$$F_2 = m_2 g$$

$$m_2 = \frac{F_2}{g}$$

$$m_2 = \frac{4N}{9.8N/kg}$$

$$\therefore \boxed{m_2 = 0.4kg}$$

2nd Demonstration (4)

- Using the same type calculation as in Example 2, we get a value of $m_2=0.4\text{kg}$
- So, we are trying to lift an object by another object almost nine times lighter...
- Can we really do this?

2nd Demonstration (5)

- We can clearly see that there is a Force-Multiplying effect
- Why doesn't the experiment work perfectly with the lighter weight?
 - We didn't account for the mass of the pistons
 - The apparatus is not completely sealed
 - There is friction present along the sides of the pistons
 - There is air trapped above the water which must compress before the water moves

2nd Demonstration (6)

- One Last Thing To Consider:
 - The smaller piston moved a much greater distance than the larger one.
 - Why?
 - Since the fluid is incompressible, the volume must be conserved.
 - Since the pistons have different areas, they must move different distances to displace the same volume of water.

Summary

- Pressure is given by $P = \frac{F}{A}$
- The pressure in a liquid depends on depth
- Pascal's Law:
 - *A change in the applied pressure on a fluid is transmitted undiminished to every point of the fluid and the walls of the container*
- Hydraulics are a practical application of these principles, and can be used to achieve force multiplication