Gyroscopic Stabilization and Angular Momentum

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UBC - Physics 420

Outline

- What is a gyroscope?
- What is angular momentum?
- How are gyroscopes used today?

What is a gyroscope?

A spinning wheel on an axle.

- Bike wheel.
- Spinning top.
- Yo-yo.

What is so special about a gyroscope?

They display a very interesting feature:

A spinning gyroscope "wants" to keep spinning in the same axis.

It resists tilting!

Why does this happen?

To answer this we need to discuss:

ANGULAR MOMENTUM

What is angular momentum?

To answer this let's review linear momentum

Momentum = Mass • Velocity

OR

$$\mathbf{p = mv}$$

REMEMBER: Momentum points in the same direction as v!

What does velocity correspond to in Angular Momentum?

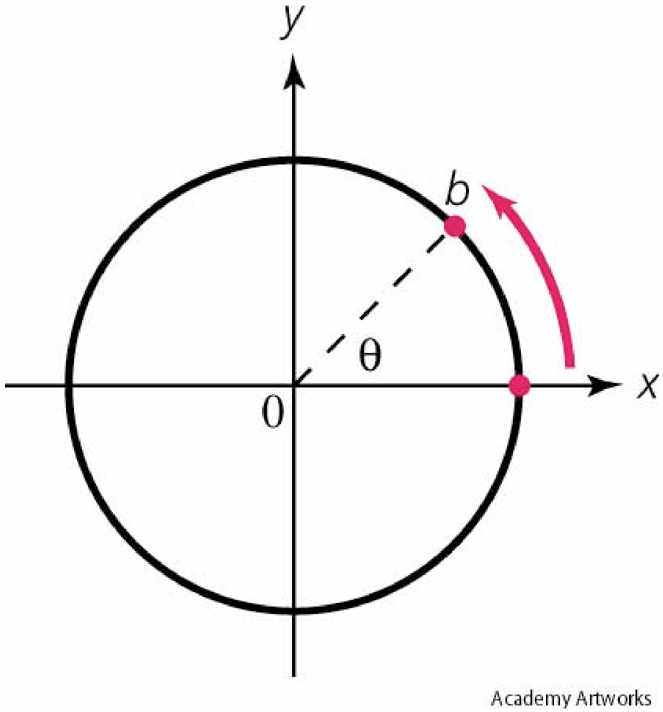
Velocity (v)

↓

Angular Velocity (ω)

What is angular velocity?

Rate of change of angle per change in time.



Hint: *Easy way to calculate angular velocity is to time how long it takes to move around the circle once. Then we can just divide 360º by the time it took to get angular velocity.*

QUIZ

It takes the earth 24 hours to make one complete rotation.
What is the angular velocity that the earth is spinning at?

QUIZ

It takes the earth 24 hours to make one complete rotation. What is the angular velocity that the earth is spinning at?

ANSWER: 24 hours = 86,400 seconds

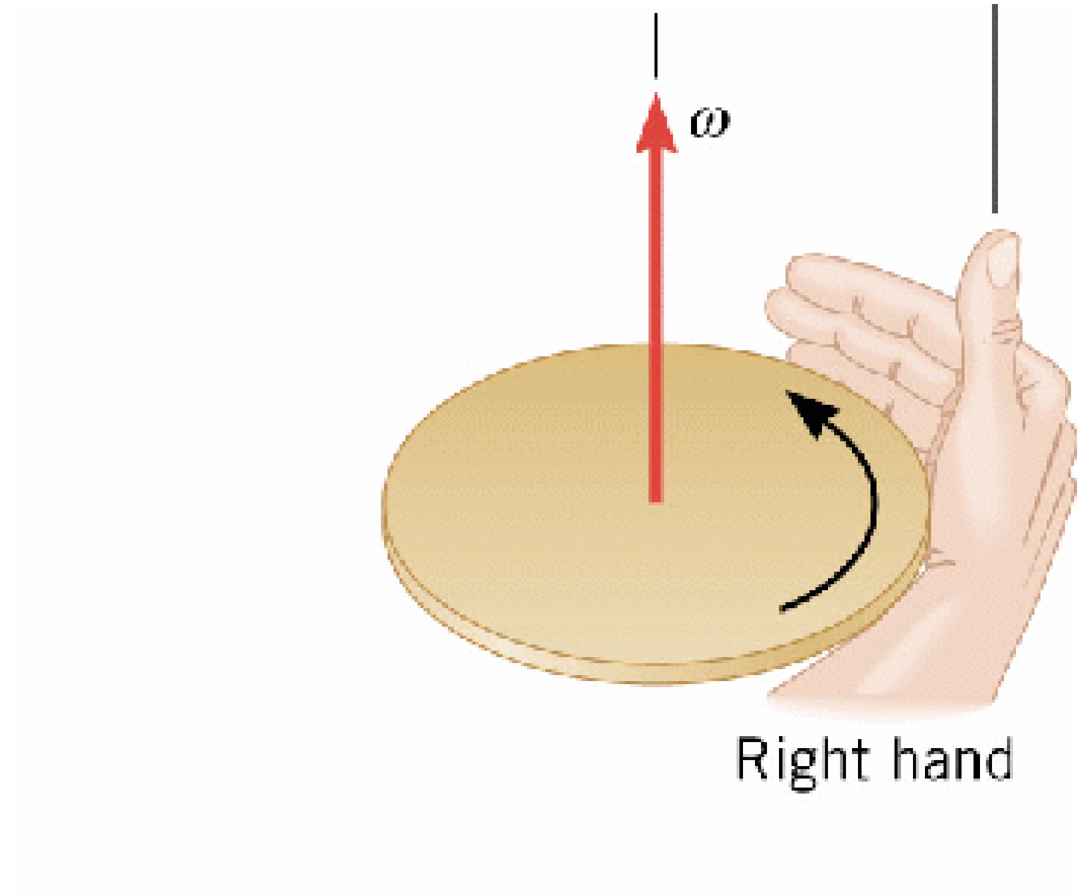
One complete rotation = $360^\circ = 2\pi$ radians

ω = change in angle/change in time

$\omega = 2\pi / 86{,}400 =$ 0.000072722 radians/s

What direction does ω point in?

RIGHT HAND RULE



QUIZ

What direction is this figure skater's angular velocity pointing in? (Remember the Right Hand Rule)

QUIZ

What direction is this figure skater's angular velocity pointing in? (Remember the Right Hand Rule)

ANSWER: **Straight up**. Use the right hand rule and you can see your thumb points up when curled along the direction of rotation (along the red arrow).

What does mass correspond to in Angular Momentum?

Mass (m)

↓

Moment of Inertia (I)

What is moment of inertia?

In angular momentum, we cannot consider a single mass moving at a velocity v. We have to consider many masses located at different distances from the axis all moving with different velocites (but same angular velocity).

When we sum up all the masses around the axis we get **MOMENT OF INERTIA (I)**

The moment of Inertia for a point mass at a distance R from the axis of rotation is:

$$I = mR^2$$

Using calculus, we add up the moments of Inertia of all the point masses of a solid object.

This has been done and the moments of Inertia for many common shapes can be easily looked up in a general physics textbook.

Angular Momentum Formula

Now that we've covered **angular velocity** and **moment of inertia**, let's use these to make an angular momentum formula.

Angular Momentum = Angular Velocity • Moment of Inertia

$$L = I\,\omega$$

REMEMBER: Angular momentum points in the same direction as ω!

QUIZ

What direction is this figure skater's angular momentum pointing in? (Remember the Right Hand Rule)

QUIZ

What direction is this figure skater's angular momentum pointing in? (Remember the Right Hand Rule)

ANSWER: The angular momentum is pointing straight up. If you curl your fingers in the direction of rotation, your thumb will point in upwards. This is the direction of the angular momentum.

Is Angular Momentum Conserved?

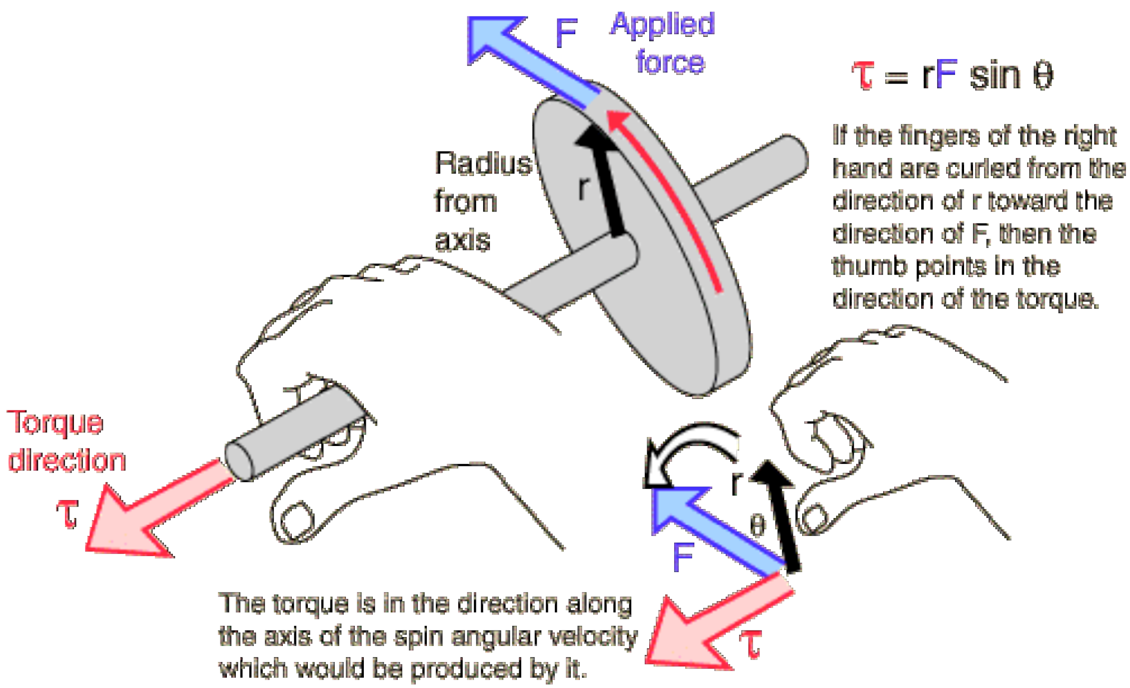
- Just like linear momentum, angular momentum obeys a conservation law.
- In linear momentum, a change in momentum results in a force

Force = Change in momentum / Change in time

$$F = \Delta p / \Delta t$$

Torque (τ)

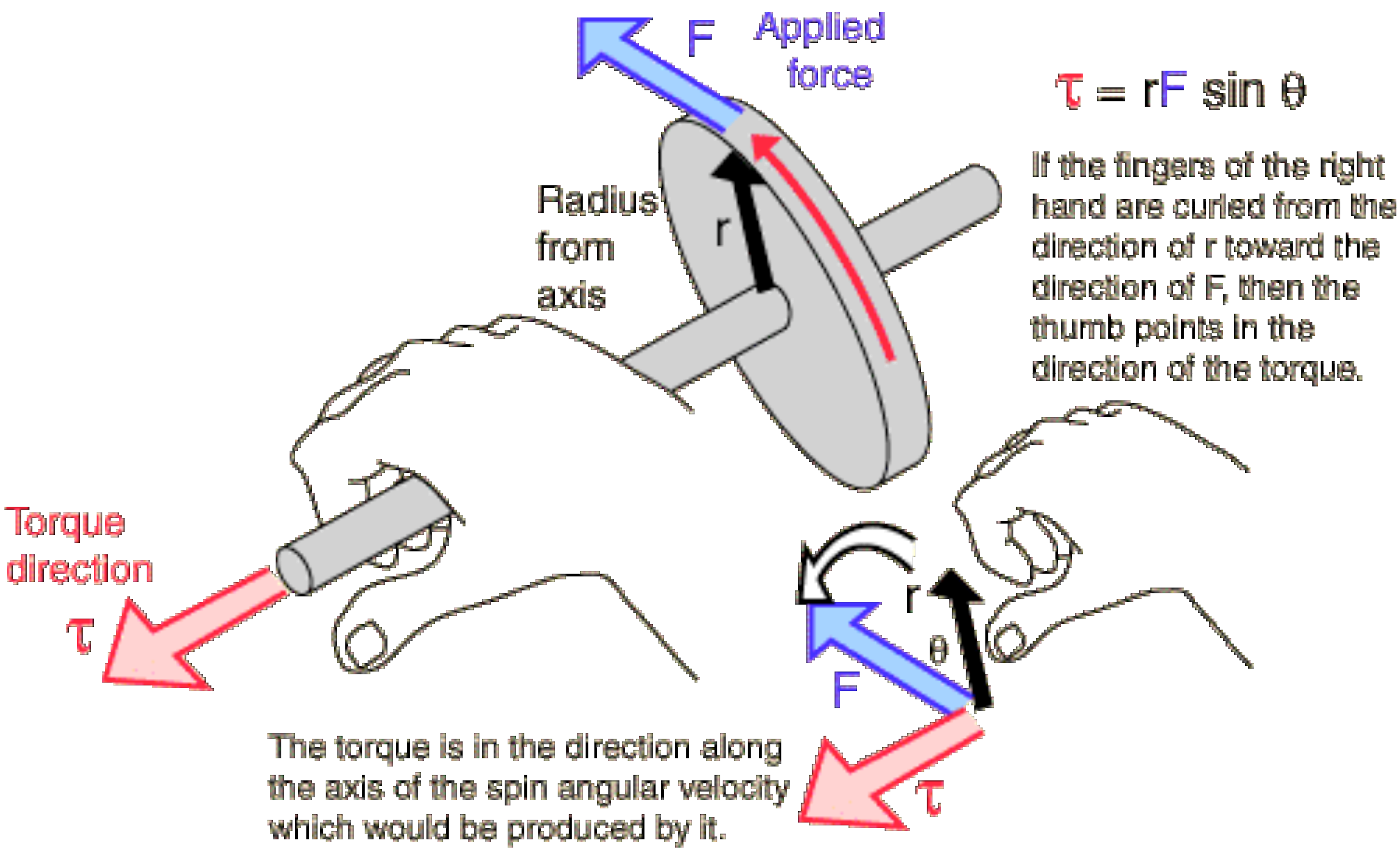
- In angular momentum, the equivalent of Force is **TORQUE**.
- Like a "rotational" force, or a force acting at a distance **r** from the axis of rotation.



Direction of Torque

- Need to use your right hand again.
- Torque is perpendicular to both the direction the force is applied and the vector from the axis to the location of the applied force.
- Take your right hand and point your fingers in the direction of r, when you curl your fingers in the direction of the applied force, your thumb point in the direction of the torque.

Direction of Torque



Changes in Angular Momentum

- Similar to linear momentum, a change in angular momentum causes a **torque**.

Torque = Change in Angular Momentum / Change in time

$$\tau = \Delta L / \Delta t$$

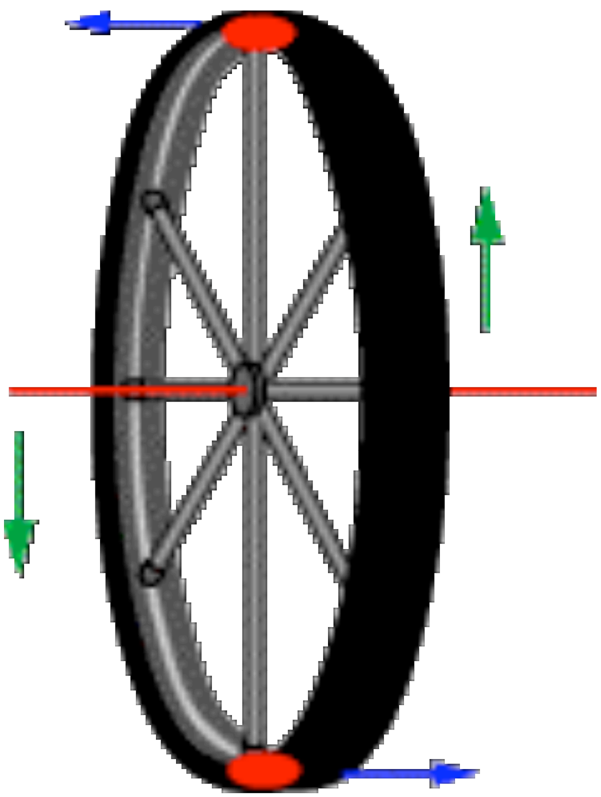
NOTE: Change in angular momentum and torque both point in the same direction!

Linear Momentum (p)	Angular Momentum (L)
Mass (m) →	Moment of Inertia (I)
Velocity (v) →	Angular velocity (ω)
Force (F) →	Torque (τ)

Let’s use what we just learned to understand why a bicycle wheel resists tilting.

QUIZ

When we tilt the wheel (shown by the blue arrows), which way is the angular momentum changing?



QUIZ

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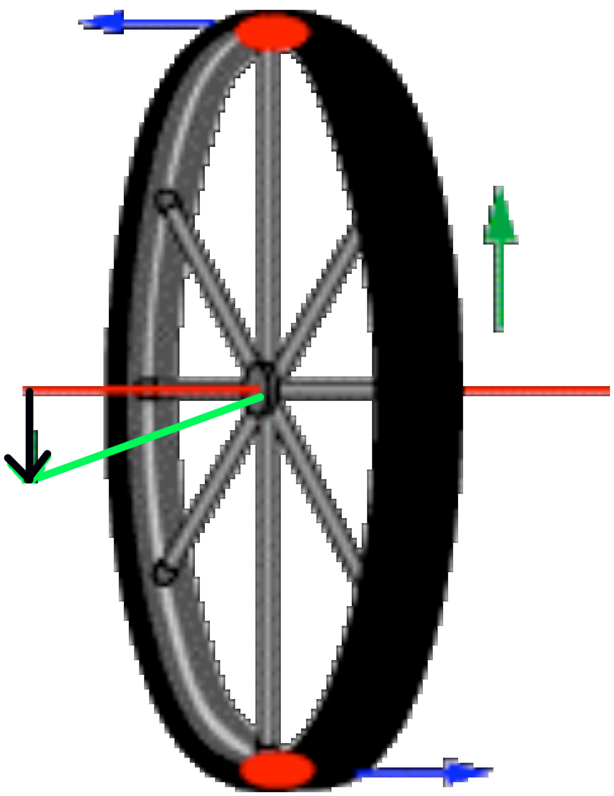


ANSWER: It changed in the direction pointing down as shown by the black arrow.

Let's look back to Torque

$$\tau = \Delta L / \Delta t$$

So from this formula we can see that the higher the change in angular momentum is, the bigger the torque we need to tilt the wheel is going to be.

SO… ***the faster a wheel is spinning, the harder it is to tilt!***

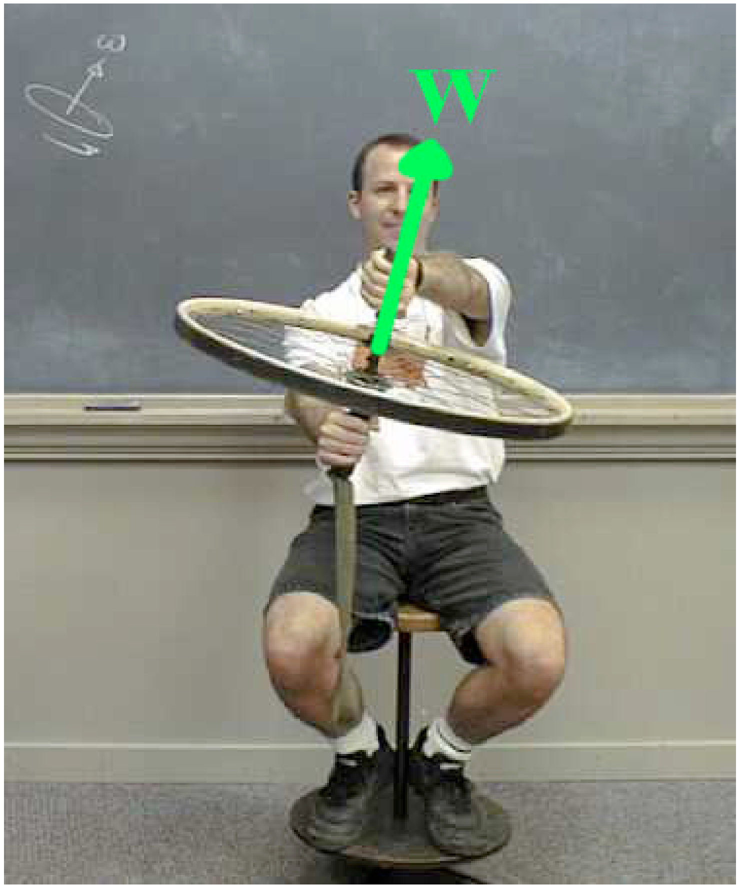
This explains why the faster we had the bike wheel spinning, the more it resisted the attempt to tilt.

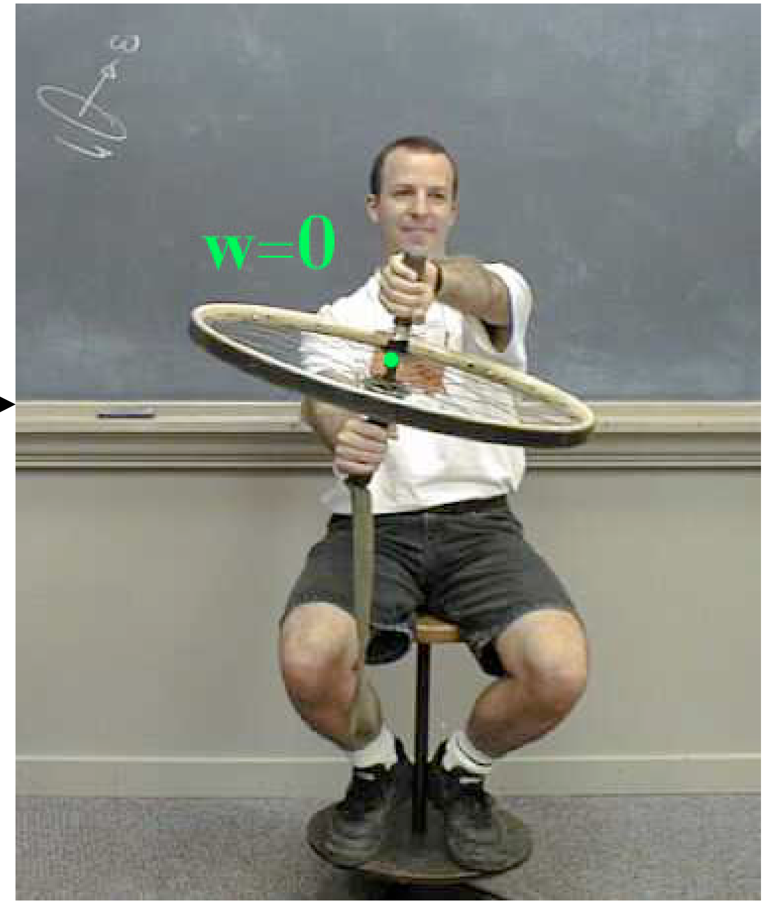
Let’s use what we learned to understand why stopping the bike wheel caused me to spin.

QUIZ

If I am holding a bike wheel horizontally (as in the original demo) and the rotation is stopped suddenly, in which direction does the angular momentum change?

QUIZ



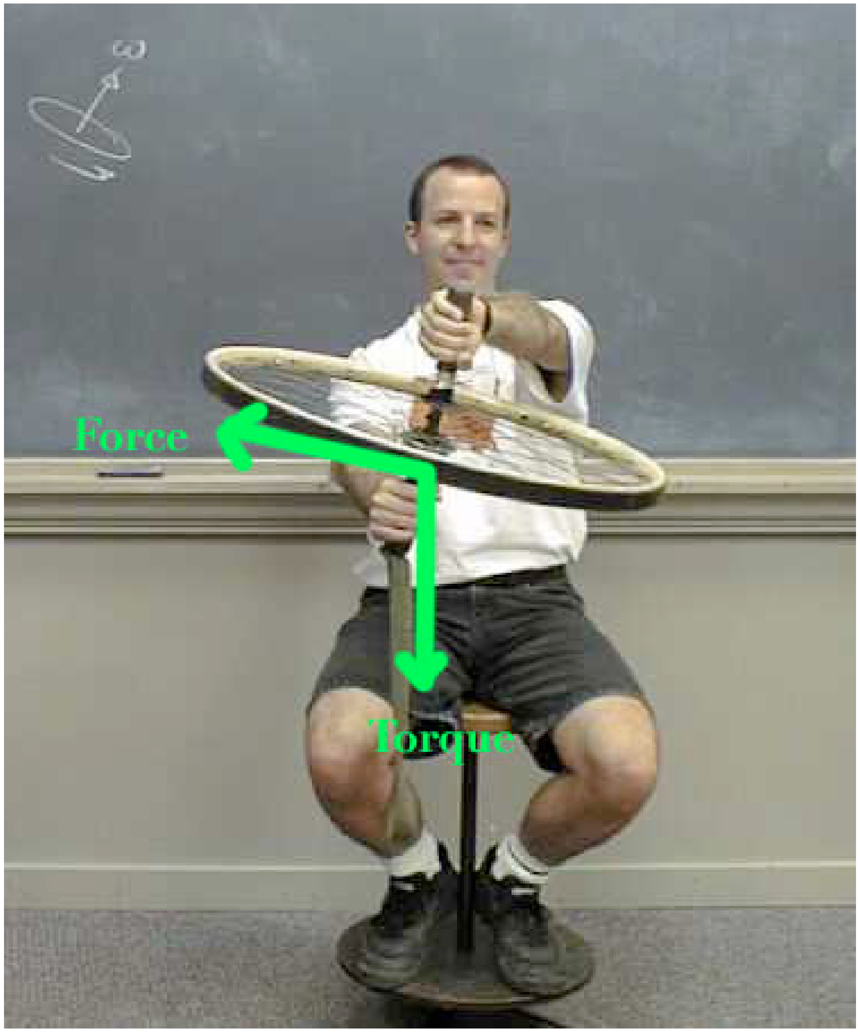


So angular velocity (and thus angular momentum) went from pointing straight up to zero. This means it had to change in the direction pointing DOWN.

What torque does this create?

Torque points in downward direction as well.

Using right hand rule this means we have a force along the edge of the wheel pointing to the left.



This explains why when I stopped the wheel on my body, I rotated on the lazy susan!

Satellites

- In space, a satellite cannot “push off” of anything so it relies on gyroscopes to control it’s orientation.
- Satellites use these 2 examples described to “control” their orientation when they are in orbit.

Stabilization

1- Rotation: the satellite adjusts the speeds of three perpendicular wheels to rotate itself in any direction it wants.

2- Holding position: the satellite uses the fact that a spinning wheel resists tilting to hold it's position in space.

END OF LESSON!

Now let’s use the model satellite to test the concepts we just learned.