Kinematics Demonstrated Through Projectile Launcher

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Physics 420: Demonstration Physics

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TABLE OF CONTENT

INTRODUCTION 2

THEORY 2

DEMONSTRATION 6

i) APPARATUS 6

ii) APPLICATION 8

CONCLUSION 10

INTRODUCTION

The field of kinematics, also known as classical mechanics, describes the motion of an object without any consideration of the events leading to the action. Even at its simplest form, kinematics brings physics to life. Not only is this subject intriguing to young students who want to see their school work reflect every day examples, but it can be solved using elementary mathematics that doesn't distract the student from the overall picture. Kinematics is one of the first branches of physics studied at the high school level. I firmly believe that by utilizing exciting teaching techniques such as relevant demonstrations in the physics classrooms, students' minds will be captivated by the phenomena of physics. Studying kinematics and experiencing first-hand the wonders and relevance of the field was the underlying motive that led me into studying physics at the university level. My goal in this course is not to build a cool projectile that is possibly capable of destructive consequences, but to show students that physics is everywhere we look and is being used daily to explain mysteries and develop technology. With that said, I decided that in order to demonstrate kinematics in a matter that stimulates and intrigues the class, I would build an air pressured projectile launcher that would fire tennis balls. The purpose of the launcher is not simply to create excitement in the classroom, but to be used as a tool itself to measure how the equations of motion compare to experimental testing.

THEORY

As previously mentioned, classical mechanics is the study of the motion of objects. Classical mechanics can be divided into two fields: dynamics (the study of force on objects) and kinematics which describes how objects move. In our case we will only be dealing with translational motion – motion without rotation. The physics behind classical mechanics is credited (due to major contributions) to Galileo Galilei and Sir Isaac Newton.

Kinematics is explained through Newton's laws – principally his first which states that an object in motion will remain in motion until it is acted upon by an external force. Gravity, friction and drag are only a few examples of external forces. For the sake of my presentation, we will only be dealing with gravity as an external force. Before we can explain and derive the equations of motion, we must first revisit Newton's contribution to the field. Aside from Newton's three laws, he also contributed to the mathematics and physics community with the introduction of calculus – specifically rates of change. We will make two simple assumptions for the equations of motion:

1) Acceleration is uniform (constant) throughout the motion, and
2) The motion is constraint to a straight line (ie. 1 dimensional).

In order to properly understand this field of physics, one must have a good grasp of reference frames. We define position of an object using Cartesian coordinates of *x* (and y for 2 dimensional space). The displacement is defined as change in position: *Δx.* We will also need to have an elementary understanding of vectors and trigonometry. All these tools allow us to separate a vector in any direction into two components in the x and the y plane (again, this is for a 2 dimensional space).

Now we can define average velocity as the total distance traveled (displacement) divided by the time it took to travel.

$$\bar{v} = \frac{distance\ traveled}{time\ elapsed} = \frac{x_f - x_o}{t_f - t_o} = \frac{\Delta x}{\Delta t}$$

We begin the derivation of the equations of motion by defining the acceleration of an object. At the high school level this could be explained using the instantaneous velocity/acceleration as above; however, I will use some calculus in order to obtain a cleaner derivation. We define the rate of change of velocity as the acceleration a:

$$a = \frac{dv}{dt}$$

Now we can use the technique of separation of variables to rewrite the equation into the form:

$$a * dt = dv$$

Now we can integrate both sides

$$\int_0^t a * dt = \int_{v_0}^{v_f} dv$$

The bounds of our velocity integrand are from the final velocity to the initial velocity (change in velocity). Similarly, the bound for the time integrand is the change in time. However, we will take the initial time to be 0. And since the acceleration is constant, we can remove it from the integral.

$$a \int_0^t dt = \int_{v_0}^{v_f} dv$$

Now integrating both sides:

$at|_0^t = v|_{v_0}^{v_f}$ → $a * t = v_f - v_0$

$$\rightarrow \quad v_f = v_0 + at \qquad (1)$$

Similarly we can work out the other equations of motion. This time we begin with the expression for velocity:

$v = \frac{dx}{dt}$ → $dx = v * dt$

And again we will utilize separation of variables and then integrate both sides.

$$\int_{x_0}^{x} dx = \int_0^t v\, dt$$

We will use the same reasoning for the bounds as we used above. Examining our integral, we can see that we already have an expression for the velocity. We can substitute $v_f = v_0 + at$ for v.

$$x|_{x_0}^{x_f} = \int_0^t (at + v_0)dt$$

$$x_f - x_0 = \frac{1}{2}at^2 + v_0 t$$

To further simplify our expression, we can define distance *d* as:

$$d = x_f - x_0$$

We have now solved for our second equation of motion.

$$\Rightarrow \quad d = \frac{1}{2}at^2 + v_0 t \quad (2)$$

We have just derived the two basic equations of motion using the definitions of acceleration and velocity. Both these equations explain the same type of motion but involve different variables. In kinematics, you will usually have an unknown variable that must be solved. In order to simplify the mathematics involved in the calculations, we will use both our equations to eliminate further variables (and in turn, creating two more equations of motion).

We can solve (1) for t:

$$t = \frac{v_f - v_0}{a}$$

We can now plug this expression into (2) and simplify the equation:

$$d = \frac{1}{2}a\left(\frac{v_f - v_0}{a}\right)^2 + v_0\left(\frac{v_f - v_0}{a}\right) = \frac{1}{2a}\left(v_f{}^2 - 2v_f v_0 + v_0{}^2\right) + \frac{v_0 v_f}{a} - \frac{v_0{}^2}{a} = \frac{v_f{}^2}{2a} - \frac{v_0{}^2}{2a}$$

$$\Rightarrow \quad v_f{}^2 = v_0{}^2 + 2ad \quad (3)$$

We have now solved our third equation of motion (which does not depend on time).

We can also solve (1) for a:

$$a = \frac{v_f - v_0}{t}$$

Similarly, we can plug this expression into (2) and simplify the equation to get our last equation of motion:

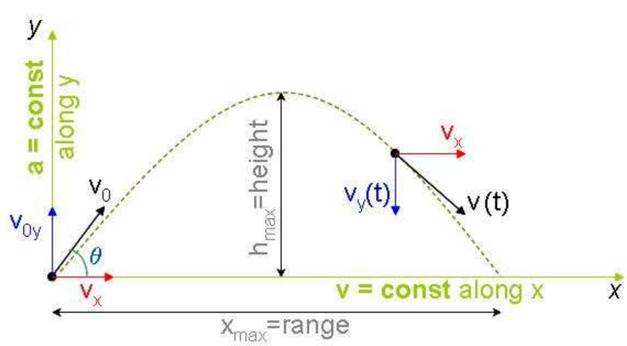
$$d = \frac{1}{2}\left(\frac{v_f - v_0}{t}\right) t^2 + v_0 t = \frac{1}{2}\left(v_f - v_0\right) t + v_0 t = \frac{1}{2} v_f t + \frac{1}{2} v_0 t$$

$$\Rightarrow d = \frac{1}{2} t\left(v_f + v_0\right) \quad (4)$$

The fourth equation of motion does not depend on the acceleration. We have now found 4 equations that can be used to solve a variety of problems.

1) $v_f = v_0 + at$
2) $d = \frac{1}{2} at^2 + v_0 t$
3) ${v_f}^2 = {v_0}^2 + 2ad$
4) $d = \frac{1}{2} t\left(v_f + v_0\right)$

It is important to understand that the reason we went as far as to derive 4 different equations is to make sure that given any unknown between the acceleration, distance, initial velocity, final velocity and time, we can solve for it using one of the above equations. These equations involve simple algebra but are powerful tools that we will use to calculate several problems.



DEMONSTRATION

i) APPARATUS

This demonstration involves building a projectile launcher. This is an air pressured “cannon” which will shoot tennis balls out of a barrel. In order to get the velocity we want out of the projectile, we must create a pressure difference. This is accomplished by building up high pressure into a 2 Liter bottle using an air pump (or possibly an air compressor). The tube connecting the pump to the bottle is a one way valve that will only allow air into the bottle (similar to how an inflatable beach ball works). The bottle's other end will be connected through more tubing to a valve and then to a barrel large enough to hold a tennis ball. There will also be a pressure gauge to measure the exact pressure in the bottle. This is important in order to get the appropriate speed out of the tube and also to not over pressurize the bottle since an explosion can be very dangerous. Once the appropriate pressure level in the bottle is reached, I will turn the valve, allowing the air to escape out of the tube. This will force the tennis ball out of the tube creating a projectile. I can vary the length of the barrel which would vary the velocity. The longer the barrel, the more time the tennis ball will have to accelerate before it leaves the barrel. The barrel will be made of transparent plastic (or similar material). The angle of the barrel will also be adjustable. Below is an illustration of the apparatus involved in this demonstration

ii) APPLICATION

Now that we have derived the equations of motion and explained the apparatus, I will use projectile motion to demonstrate their application. We are now branching a bit into the dynamics side of classical mechanics. The reason I chose projectiles is that I believe that their relevance to real world application (such as a baseball being tossed to a teammate or an airplane dropping a missile) are captivating to students.

As mentioned throughout the derivation, although we are dealing with constant acceleration of the object in motion, the object will not necessarily be unaffected by external forces. Although gravity will not affect horizontal motion, it will affect any type of vertical motion (y component of the motion). This means that we will need to take the gravitational force into account for all projectile questions. Gravity, at the surface of the earth, has acceleration:

$$a_g = g = -9.8\ m/s^2$$

The negative sign denotes that the force acts downwards.

As a projectile is fired at angle θ, we will take several steps into predicting its motion. Firstly, we will separate its initial velocity into x and y components. This way, we can deal with each direction independently, and then add them up at the end. The horizontal component is simple since it is not being affected by gravity. We can then use the simply equation of

$$v_x = \frac{distance}{time} = \frac{d}{t}$$

In the vertical direction, most of the time all we will be looking for is the time the hang time – the amount of time the object will stay air born until gravity not only slows it down to a halt, but brings it back down. We can have a variety of questions regarding projectiles but they will all be solved in similar manners. I will reiterate the importance of separating vectors and being able to put them back together.

Since the projectile launcher is to be used as a tool in teaching the class and not only as a final demonstration, it is important to understand how it works. From definition kinematics does not in into account the means by which the motion occurred, but simply that it is occurring. However, I will need to know what the velocity of the projectile is as it is fired out of the launcher. We will calculate the initial velocity using two methods: experimental and theoretical.

1) **Experimental:** This will be done once the fabrication of the projectile is complete. I will fire the projectile, measure the time and distance it traveled, then calculate its initial velocity. Also, I would like to calculate using a radar gun or similar apparatus to get an even more accurate result.
2) **Theoretically:**

$Force = Pressure\ x\ Area$, but from Newton's 2nd Law we also know that $F = mass\ x\ acceleration$

$$a = \frac{P * A}{m} = \frac{\Delta P * \pi r^2}{m}$$

This is the acceleration of the ball. However, the ball will only accelerate throughout the length of the tube.

$$v_f^{\ 2} = v_0^{\ 2} + \ 2 * a * d$$

Plugging in the earlier expressing for the acceleration, and knowing that $v_0 = 0$, we get the velocity v_f of the ball leaving the tube.

$$v_f = \sqrt{2 * \frac{\Delta P * \pi r^2 d}{m}}$$

This is important for me to know, however it does not necessarily have to be understood by the class for the presentation. The class only needs to [illegible]w the velocity of the projectile when it leaves the barrel. It is also important to note that there will surely be discrepancies between the experimental result and the theoretical result. This is simply due to the fact that theoretically we have ignored many variables such as friction and drag. For example, in an ideal situation (theoretical), it will take a pressure difference of 2900 Pa in order to fire the tennis ball at a velocity of 14 m/s (which corresponds to a displacement of 10.0 meters fired at an angle of 45^{o}).

As formally mentioned, the purpose of the demonstration is to be used as a calculation tool during the lesson. In doing so the class not only experiences the presentation but also gets to apply their knowledge to predict and then measure the results. First, we will examine several questions. These questions have not yet been set in stone but will most likely be:

1. A simple projectile launched from level ground at an angle θ where we would calculate the total distance traveled.

2. A projectile launched off a platform of height x (to be determined), which would fly through the air and land on a lower surface (height x=0). Again we would be calculating the total distance traveled.

ieve that this demonstration is unique since it is a tool to be used by the students, not just a presentation that they watch from their desk. It is very important in this experiment to use accurate significant figures since there is bound to be a certain amount of error. It is even more important to explain the reasons behind the error. Physics is very simple on paper its ideal form; nevertheless, in practice there are forces which cannot be ignored and factors that must be corrected.

CONCLUSION

Now that I have outlined the work involved in fabricating and effectively demonstrating the principles of kinematics through a projectile launcher, it is evident that time is of the essence. For the purpose I propose the following time line for the fabrication and preparation process:

- Examine all materials available at the machine shop and consult with the machinists. Their experience will be a good input as to what might need to be changed or adjusted to achieve a successfully demonstration.
  Date: November 7th, 2008
- Gather material and begin fabrication. Ideally, this should be put together by the machine shop. Their professionalism and scrutiny to detail is a great asset.
  Date: December 1st, 2008
- Test the projectile and evaluate the required velocity. It is important that the launcher is put to excessive testing so it is ready (and flawless) during the school presentations.
  Date: January 5th, 2009

The final aspect that I must take away from this report is the possibility of error throughout this project. There are a lot of assumptions that are made in the equations of motion. Unfortunately, in real like you cannot ignore friction or drag. It is not only important that I take these factors into account, but that I relay this information to the class and teach them the importance of the application of physics.