Stirling Engine

1. Introduction

In secondary schools the fundamentals of kinematics and some electromagnetism are taught in lectures and demonstrated in laboratories. There is likely to be less emphasis on thermodynamics and concepts that are covered in lectures are probably not demonstrated. The student are of the age where they are new drivers and perhaps interested in engines. Moreover, there is a lot of recent interest in the efficient use of energy in today's world. The Stirling engine can be a very good tool for demonstrating some very basic and fundamental physics while discussing topics that are likely to attract the attention of the students. This paper will: (1) give a simple discussion of the mechanisms that make the Stirling engine work, (2) give an overview of the concepts that can be learned from the engine, (3) give a rough outline an example lesson plan, (4) discuss what needs to be built, and (5) sketch a plan for the next two months.

2. What is a Stirling Engine ?

A Stirling engine is one example of a broad class of heat engines which are devices designed to convert thermal energy into mechanical motion. The internal combustion, or gasoline, engine in an automobile is another example of a heat engine [1]. The gasoline engine uses the combustion of fuel inside a confined volume, whereas the Stirling engine uses an external heat source to heat the working substance. The heat source can come from burning fossil fuels (such as gasoline), solar energy, decaying plant matter, or whatever is available [2]. In fact, all the Stirling engine requires to operate is a temperature difference. It is possible to run a Stirling engine by cooling one region of the engine below ambient temperature. The gas inside the cylinder of a Stirling engine is not burned or consumed in anyway. Hence, in contrast to the internal combustion engine, the Stirling engine does not require an exhaust or an intake. If a clean (green) external heat source is used with the Stirling engine, it can be an environmentally friendly alternative to engines that burn and emit hydrocarbons and other pollutants. Stirling engines also limit noise pollution because they do not require intake and exhaust valves which usually are the main source of engine noise. However, Stirling engines that would be suitable for automobile use are larger, heavier, and more expensive than conventional internal combustion engines. Moreover, Stirling engines require some time to warm up before they starts and the output of the engine can not be changed quickly for quick acceleration and deceleration. Although Stirling engines have not yet found use in the automotive industry, they have been used as a submarine engines. Recently, there has been a resurgence of interest in Stirling engines as the demand for more fuel efficient and clean engines continues to increase [2, 3].

3. How does the Stirling Engine Work ?

The Stirling engine operates by repeatedly completely a sequence of four steps. Each step in the sequency is reversible and together they form the *Stirling cycle.* To help understand each of the four steps in the Stirling cycle consider two gas filled cylindrical pistons whose chambers are connected by a narrow tube as pictured in Figure 1 [3].

The left piston is at temperature T_H and the right piston is at temperature $T_C < T_H$. In the centre of the tube that connects the two chambers is a wire mesh that will be used to temporarily store heat as described below. For each step in the Stirling cycle the schematic diagrams of Figure 1 will be mapped to curves on a pressure-volume plot of the Stirling cycle shown in Figure 2. The four steps of the idealized Stirling cycle are [3]:

(1→2) The gas in the engine is expanded at the constant temperature T_H. The left piston moves down and the right piston is fixed. In ordered to maintain a constant temperature the gas must absorb heat Q_H from the reservoir (Isothermal expansion - Figure 1a, path 1→2 in Figure 2).
(2→3) At constant volume V_2, the temperature of the gas is reduced from T_H to T_C. The left piston is compressed and the right piston expanded so that the total volume remains fixed. The hot gas is forced from the left chamber to the right chamber. As the gas passes through the narrow tube it delivers heat Q to the wire mesh. (Constant volume heat removal – Figure 1b, path 2→3 in Figure 2)
(3→4) The gas is compressed at constant temperature T_C. The right piston is compressed and the left piston is fixed. To maintain a constant temperature the gas releases heat Q_C to the thermal reservoir at T_C. (Isothermal compression – Figure 1c, path 3→4 in Figure 2).
(4→1) At constant volume V_1, the temperature of the gas is increased from T_C to T_H. The left piston is expanded and the right piston compressed so that the total volume remains fixed. The cold gas is forced from the right chamber to the left chamber. As the gas passes through the narrow tube it recovers the heat Q stored in the hot wire mesh. (Constant volume heating – Figure 1d, path 4→1 in Figure 2) [3].

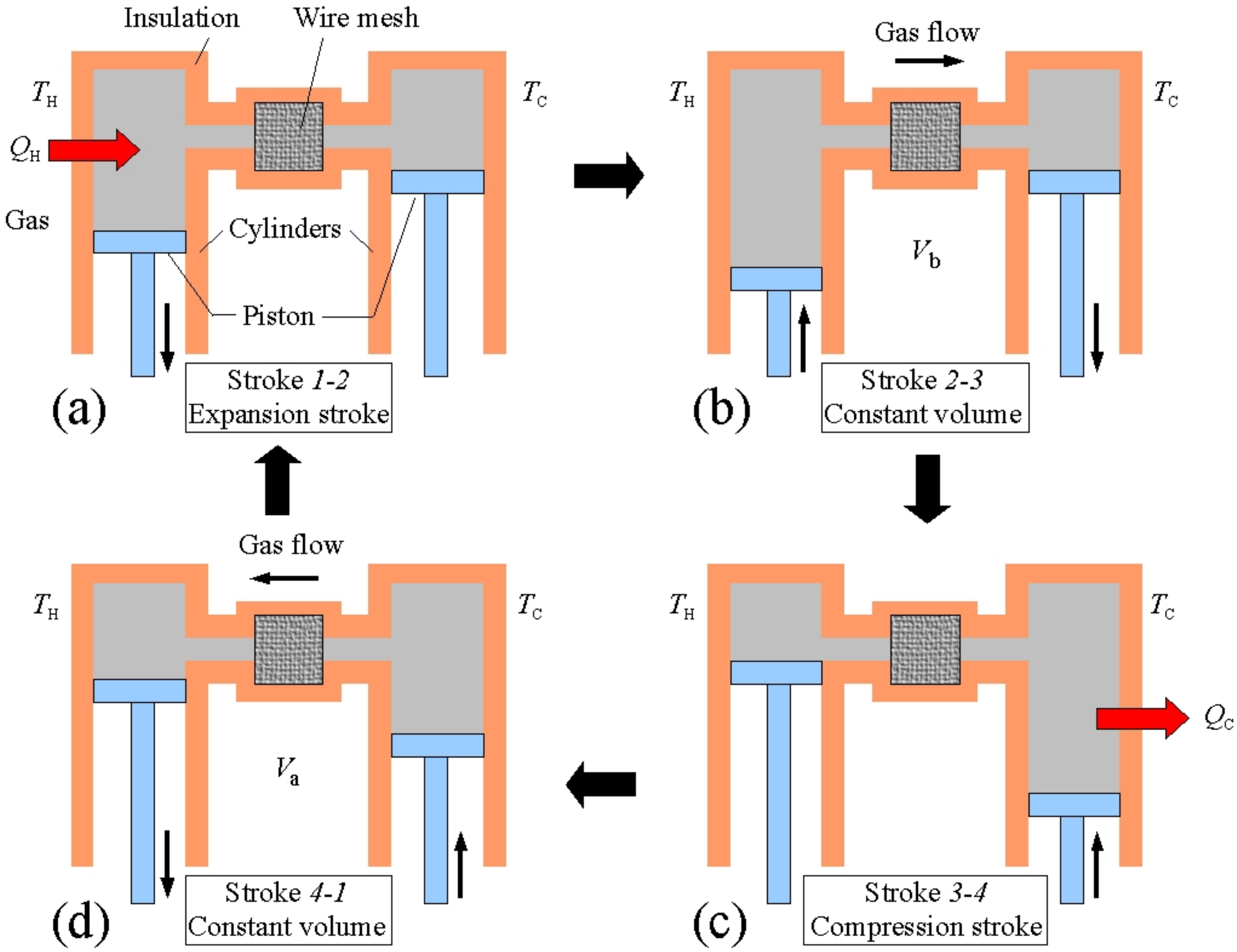

Figure 1: Dual piston Stirling engine at four different stages of the Stirling cycle.

Figure 3 schematically shows the operation of a practical (and commercially available) Stirling engine at various stages of the Stirling cycle. In any real Stirling engine the idealized Stirling cycle cannot be achieved. The four steps are blurred together and the cycle on a *PV*-diagram appears elliptical. This displacer-style Stirling engine will be the type purchased, modified, and then demonstrated to the high school students. This type of engine has one small sealed piston, called the power piston, and one larger loose fitting *displacer* piston. The role of the displacer piston is simply to move, or displace, working gas in the engine back and forth between a heated lower region and the upper cooled region. In the design pictured in Figure 3, the lower plate is heated with a flame and the upper plate is cooled by water or the ambient surroundings. The two pistons and linked together such that their movements are 90^o out of phase. That is, when the power piston is either at its maximum or minimum height and moving slowly, the displacer piston is at its halfway point and moving at its maximum speed. At position 1 of Figure 3, the displacer piston is in the upper cold region which forces the working gas to occupy the hot region and be at temperature T_H. Heat is added to the gas and it expands forcing the power piston to move upwards (path 1→2 in Figure 2). At position 2 the power piston is at its maximum height (the gas has its maximum volume V_2) and is moving very slowly approximating the constant volume path 2→3 in Figure 2. The displacer, on the other hand, is moving into the hot region causing the gas to move to the cold region. In this design, the displacer itself plays the role of the wire mesh of Figure 1 by temporarily storing energy taken from the gas as it cools from T_H to T_C. At position 3, because all of the gas is in the cold region, it contracts (heat is removed from the gas) causing the power piston to slide down (path 3→4 in Figure 2). At position 4, the power piston is fully compressed (minimum volume V_1) and is moving slowly. The displacer piston is moving upwards forcing the gas into the hot region. As the cool gas passes by the displacer it recovers the heat that was temporarily stored in the displacer (path 4→1 in Figure 2). At the completion of this process the state of the Stirling engine returns to 1 and the cycle repeats indefinitely [3].

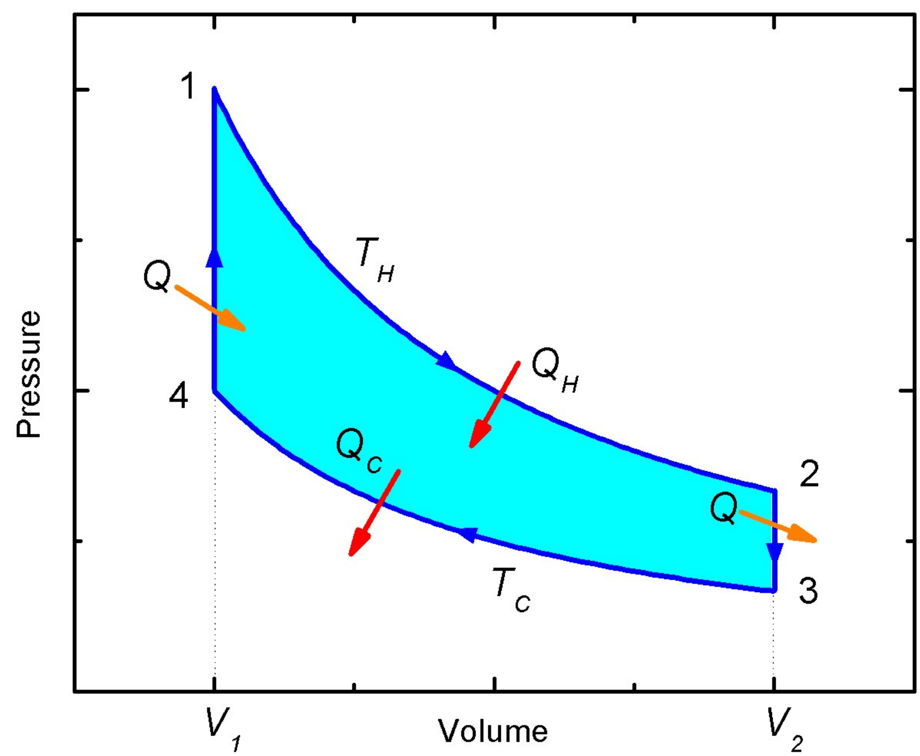

Figure 2: Pressure verses volume (PV-diagram) of the Stirling cycle.

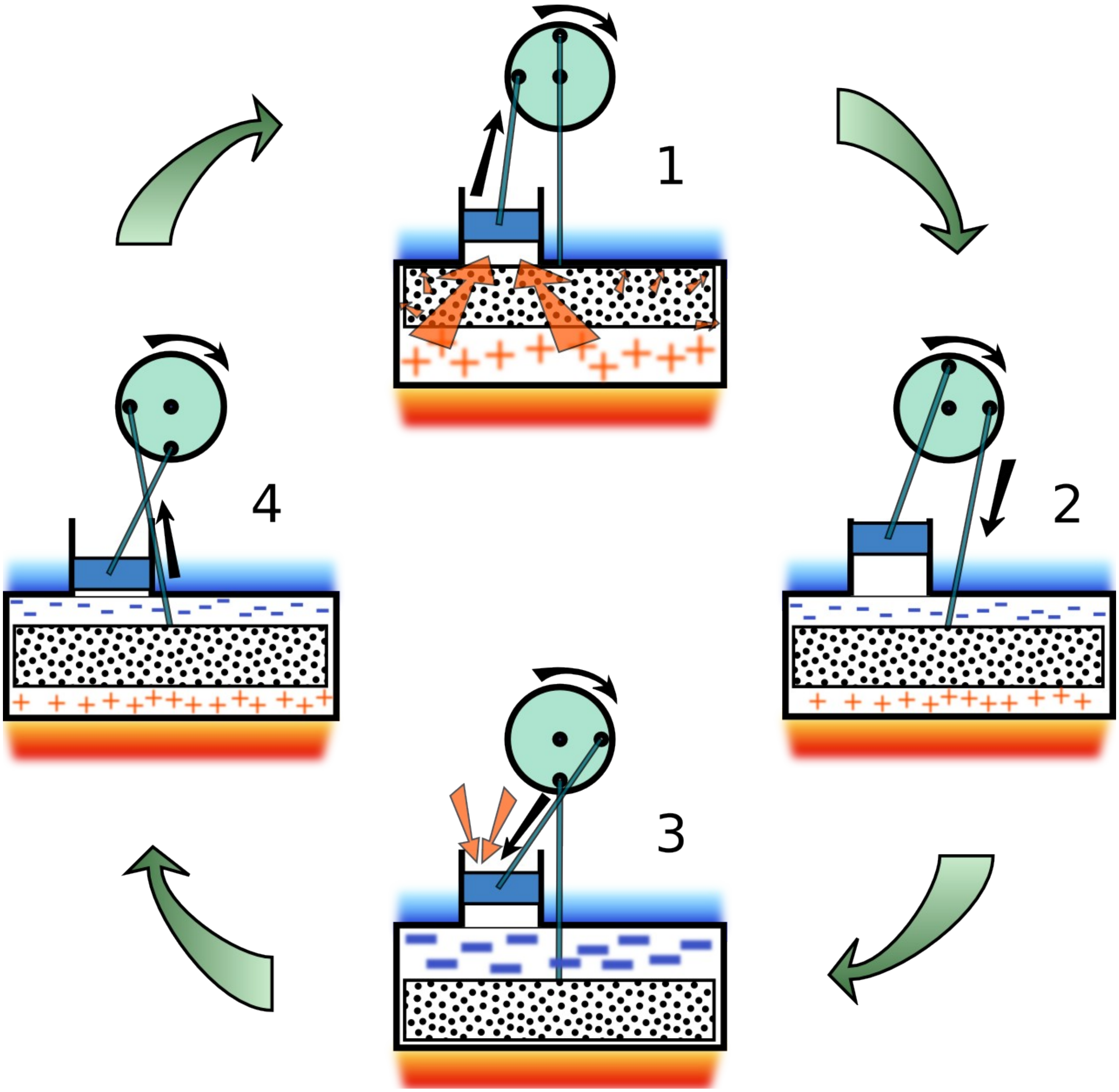

Figure 3: The different stages of operation for the common displacer-style of Stirling engine.

4. Which Physics Concepts can be Learnt?

The Stirling heat engine is very rich with physics. At first glance the explanation of the Stirling engine and Stirling cycle may appear simple, but as one investigates the details more carefully many nontrivial complexities arise. The Stirling engine also has the benefit that enthusiastic students can attempt to design and build working engines which can vary from simple to very complex.

Heat, Temperature, and Work

Heat, *temperature*, and *work* are the most fundamental concepts that can be learnt from the Stirling engine and cycle. It is essential to emphasize the difference between the meaning of heat and temperature. Heat is energy transferred from one body to another body due to a temperature gradient and temperature is simply describing the state of the matter, if it is cold or hot [1].

For the explanation of work, it is simplest to begin by defining work as the product of force on a body and the distance traveled by that body. Then, a cylinder with a cross sectional area A that contains a gas exerting the pressure P on a piston can be used as an example of work in a thermodynamic system. The force F acting on the piston due to the gas is $F = P A$. When the piston moves a distance dx due to the force F, the work done by the gas $W = Fdx = PAdx$ where $Adx = dV$ is the change in volume. Although dV is the infinitesimal volume change, dV can be expressed as $\Delta V = V_2 – V_1$ for students in secondary school. $W = P\,\Delta V$ is work done in the volume change from V_2 to V_1 at constant pressure P. After explaining these concepts the PV-diagram for a constant pressure process can be shown to point out that the area under the curve is equal to work. As seen on the left of Figure 4, for constant P, the area underneath the curve is a simple square and is equal to $P\,\Delta V$ as expected. Then, several kinds of thermodynamic processes can be shown in PV-diagrams to show different shapes of area possible while stressing that the area under the curve is always equal to the work done regardless of the shape of the curve. It is important to point out that a larger area under the curve in PV diagram indicates more work and a smaller area means less work [1].

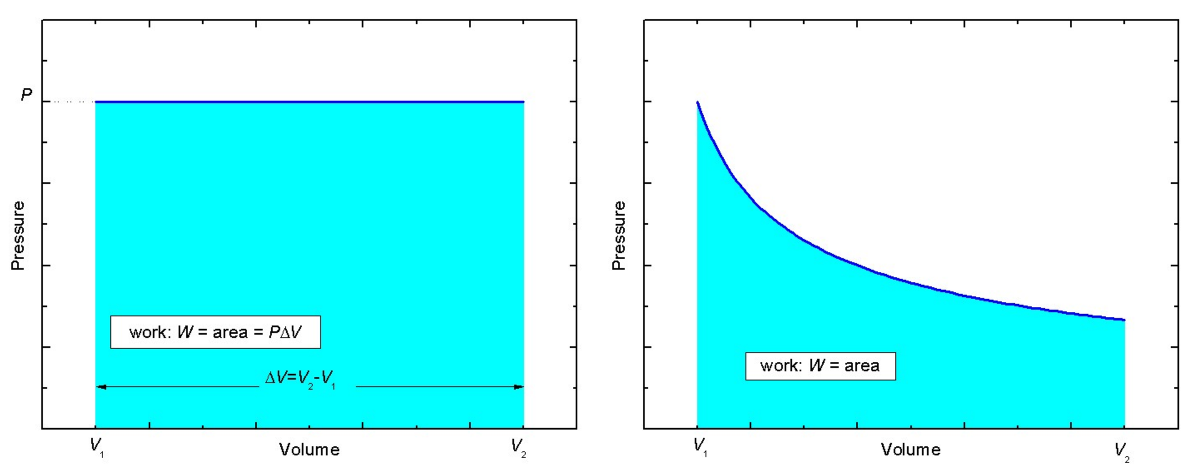

Figure 4: In PV-diagrams the work is equal to the area underneath the curve.

The First law of Thermodynamics

Internal energy of a gas can be introduced from the microscopic point of view. Matter is made up of atoms and molecules and they have kinetic energies and potential energies. The internal energy includes both kinetic energies of all particles and potential energy of the interactions between them. One can make the key point that the potential energy between the matter and the surrounding environment is not included in the internal energy [1, 3].

Then the first law of thermodynamics can be introduced by examining two situations. In the first situation, consider adding heat to a system of fixed volume (that is no work is done). Then the internal energy increases by an amount equal to heat added, $\Delta U = Q$. The second case to consider is a gas expanding and pushing a piston. In this case, if no heat is added to or removed from the system, the internal energy of the system decreases by an amount equal to the work done by the system: $\Delta U = -W$. When both cases are considered simultaneously, the internal energy is written as $\Delta U = Q - W$. This equation is the first law of thermodynamics. It should be emphasized that this law is based on careful experimental observation. From my undergraduate education I found that it can be easy to forget which concepts are based on observation and which are derived consequences. Next, three illustrations can be made to show processes for positive, negative, and zero change to the internal energy. Now, since the heat can be written as $Q = \Delta U + W$ the internal energy of the system is related its temperature, I can restate that heat can be transferred into or out of the system both by a temperature gradient and by work done by the system [1, 3].

Heat Engines

Heat engine is introduced as a device that converts the thermal energy into mechanical motion as it repeats a fixed sequence of steps. A schematic energy flow diagram can be shown for a common heat engine that operates between the hot (T_H) and cold (T_C) reservoirs. For each cyclic process, heat Q_H is added from the hot reservoir (T_H) to the engine and the engine does work W by using that heat. Not all of the heat Q_H is converted to work and the left over energy $Q_C = Q_H - W$ leaves the engine and is dumped into the cold reservoir at T_C [1, 3].

Thermal Efficiency

For each loop of the Stirling cycle, the work W done by the engine is the useful output when heat Q_H is added to the engine. In a perfect heat engine, all heat added to the system would be converted to work so that $Q_C = 0$. However, a perfect heat engine is not even theoretically possible (a consequence of the second law of thermodynamics) and Q_C never becomes zero, there is always rejected heat from the engine. It makes sense to define thermal efficiency of a heat engine as $\epsilon = |W|/|Q_H|$. W is energy we get out of the engine and Q_H is energy we pay for, or put into the engine. Because heat engines operate as a cycle, the initial and final internal energies for each complete cycle must be equal. Thus, from the first law of thermodynamics, $0 = |Q|-|W|$ where $|Q|$ is the net heat transferred and is given by $|Q_H|-|Q_C|$. Therefore, the efficiency of an ideal Stirling engine (or Carnot cycle) is: $\epsilon = |W| / |Q_H| = 1-|Q_C| / |Q_H|$ [1, 3].

Thermal Efficiency from the PV-Diagram

Now PV-diagram of the Stirling engine shown in Figure 2 can be examined in more detail. The key points are that during expansion along path 1→2 there is no temperature difference so that the change in internal energy $dU = 0$. From the first law of thermodynamics heat added from the hot reservoir is the same as the work done by the system, $Q_H = W_H$. Similarly, during the compression along path 3→4 there is also no temperature gradient so that $Q_C = W_C$. Therefore, the efficiency of an ideal Stirling engine is given by $\epsilon = 1 - W_C/W_H$. As stated above, the work W_C and W_H are given by the area under the curves $P = nRT_C/V$ and $P = nRT_H/V$ of the isothermal processes. It is difficult to obtain the area without resorting to calculus, however a simple geometric argument can be used to show that if area under the curve $y = 1/x$ is A, then the area under $y = k/x$ is kA. For the PV-diagram of the ideal Stirling engine if there is a curve $P = 1/V$ and the area under that curve is A, then the curve representing the isothermal expansion $P = nRT/V$ will have an area of $nRTA$. Thus, the thermal efficiency of the ideal Stirling cycle will be:

$$\epsilon = 1 - W_C / W_H = 1 - nRT_CA / nRT_HA = 1 - T_C / T_H$$

This equation gives the thermal efficiency of an ideal Stirling engine operating between temperatures T_H and T_C. This is the same efficiency obtained from an ideal Carnot cycle and is theoretically the maximum efficiency that can be obtained from any heat engine. It is clear that to increase the efficiency one should either use a very large T_H or a very low T_C [3-5].

5. Rough Lesson Plan?

10 minutes
The lesson will start with some introductory comments about engines in general. I will then briefly demonstrate a working Stirling engine in front of the class (place a commercially bought engine over a cup of hot water). Students are asked to think about how the engine might work. Then the Stirling engine will be briefly introduced. I will also discus the concepts of power output of an engine and engine efficiency. Next the outline of the presentation will be clearly stated. I will state the the goal of the demonstration and the concepts that will be learned. The goal is to calculate (1) the work done by Stirling engine per cycle (2) the power of the Stirling engine (work per cycle/period of one cycle), and (3) the thermal efficiency of Stirling engine. We will then measure these quantities for the real Stirling engine.

25 minutes
The concepts discussed above are explained using a power point presentation because it allows me to use many pictures making it easier for students to visualize the concepts. At least one simple question is asked for each concept. For example, students can be shown a series of PV-diagrams and are asked to determine which of the series does the most work (Fundamentals of Physics – page 467 [3]).

15 minutes
After explaining PV-diagrams and thermal efficiency, the displacer-style of Stirling engine is quickly explained. Then, the engine is run with a digital pressure gauge and a digital gauge that measures the volume so that an experimental PV-diagram of the real engine can be recorded by a laptop computer and displayed using a projector. By using an available data acquisition program the area inside the curve can be easily calculated. Having recorded the PV-diagram, it will be straight forward to calculate

the work done per cycle, the power output, and the efficiency of the engine. The students will see, for example, that the efficiency of the real engine is far less than that of the ideal engine. They will be asked to think of reasons why this might be and how the engine might be improved.

6. What do I Build?

A displacer-style Stirling engine, like in Figure 3, has been ordered. A hole, approximately 5 mm in diameter, will be made in the top (cold) plate of the engine. A plastic or metal tube with an outer diameter matched to that of the hole will be epoxied into the hole. This will need to be an air tight seal that can withstand moderate temperature gradients (Stycast 2850 or 1266 are both suitable choices). The open end of the tube will be coupled to a digital pressure gauge to monitor the pressure inside the Stirling engine. The pressure gauge will need to have a voltage output that is proportional to the measured pressure. That voltage will be acquired using Labview or Vernier. To construct a PV-diagram I will also need to monitor the volume inside the engine. Knowing the radius of the Stirling engine's power piston, I can calculate the change in volume by measuring the change in height of the power piston. I will need an instrument that can measure changes in height and again provide an output voltage that is proportional to measured height. The height and pressure will be recorded (and displayed) simultaneously and frequently updated by Labview. The program will also record time so that the period of one cycle of the engine is known. After acquiring the PV-diagram and the period the program will, in real time, calculate the work, power output, and efficiency of the engine used in the demonstration.

7. Schedule

November 01 - 15
Learning how the data acquisition program can record data from a pressure gauge and a displacement sensor. Figure out which electronic devices are required (will require two multimeters or an oscilloscope that can communicate with Labview). Order any inexpensive devices that are needed.

November 16 - 30
Work on a program to calculate the area under curves in PV-diagrams (to extract work done by the engine). Ordered kit should be arrived by this time.

December 1 - 20
Assemble all parts and measure a PV-diagram.

November 01 – December 20
Work on and refine lesson plan/power point presentation.

8. References

[1] H. D. Young, R.A Freedman, University Physics 10th ed. (Addison-Wesley, New York, 2000)
[2] http://www.howstuffworks.com/stirling-engine.htm
[3] D. Halliday, R. Resnick, J. Walker, Fundamentals of Physics 5th ed Part 2 (John Wiley&Sons, Inc, 1997)
[4] L.Viglietta, Phys. Educ. **25**, 317 (1990)
[5] C.G. Deacon, R. Goulding, C. Haridass, and B. de Young, Phys. Eudc. **29,** 180 (1994)