Propulsion and Air Resistance in Projectile Motion
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Overview: This demonstrations goal is to introduce the students to two aspects of projectile motion not usually covered: The propulsion of an object, and the air resistance it will encounter in flight. These aspects are often left out due to their not so simple applications. I will be using as simple a model as is reasonable to try to introduce the students to these concepts.

Apparatus: The apparatus used in this demonstration will be an air pressure cannon. The cannon has already been built by a previous student, I will be making slight modifications to it. The materials used in the air cannon are:
1)   Bike pump with pressure gauge
2)   Hard plastic tubing (around 2 inches in diameter)
3)   A release Valve
4)   A power source and trigger for the release valve
5)   Softer plastic tubing to connect these pieces
6)   Metal sheeting for the mount and angle adjuster
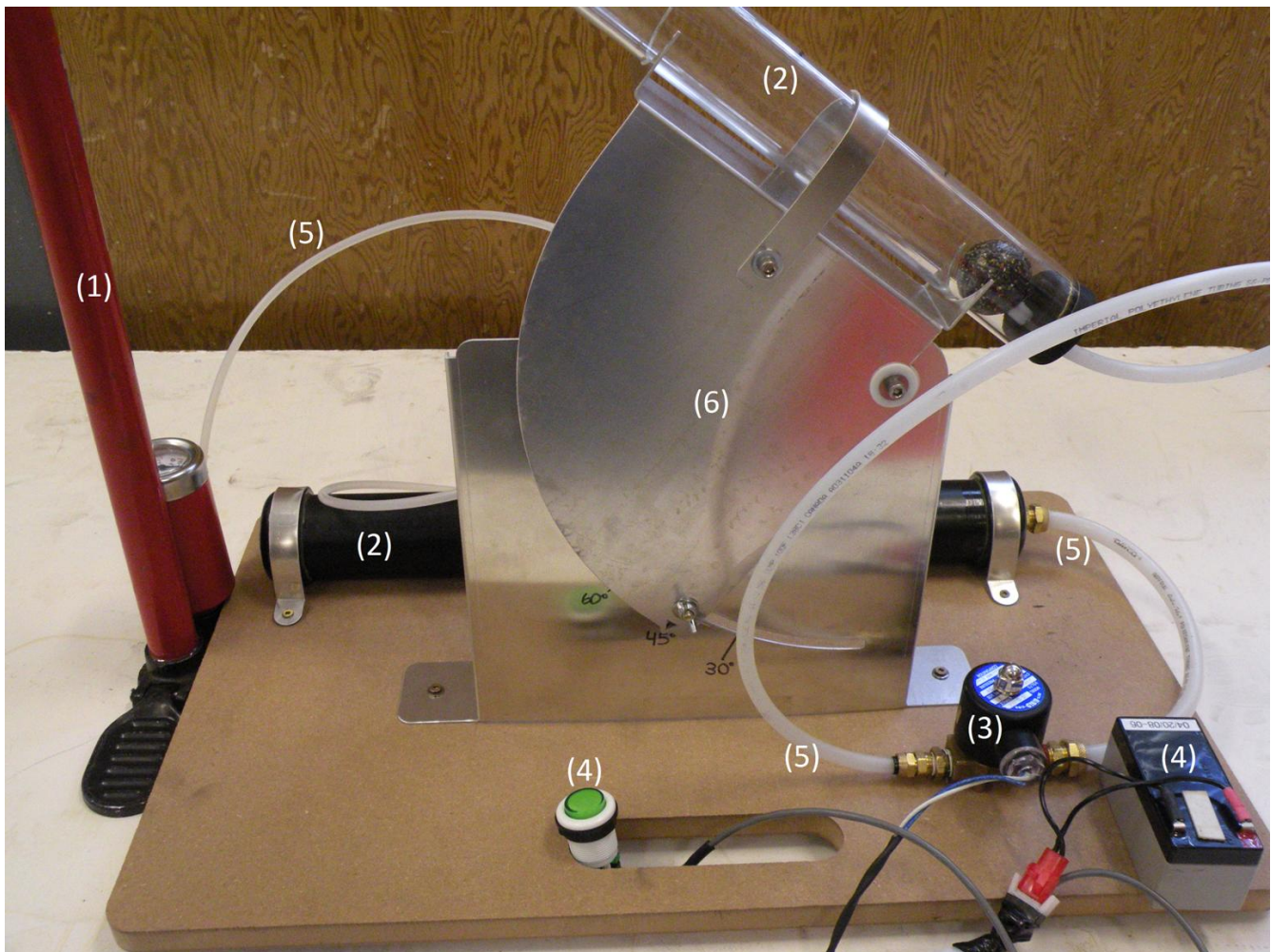7)   ~40mm Diameter O-ring

Photo Credit to Elie El-Zammar

I will be making 2 major modifications to it, and this will be done to the barrel. As is, the cannon is designed to shoot a Nerf ball, I plan to shoot a ping pong ball instead. The main purpose behind this decision is to increase the effect the air resistance will have. The change that will be made to the cannon is that I will be reducing the diameter of the barrel to something closer to that of a ping pong ball. This, by itself will not be enough however. Due to the rigid nature of the ping pong ball, a lot of the pressurized air would escape through the tiny cracks between the ball and the barrel.

To address this, I first will cut out the final 2cm or so of the barrel. I will then, using a Dremel tool to create 2 grooves. One will be on the inside part of the 2cm part we just cut off. The other will be on the barrel. These grooves will be deep enough such that when the last 2cm of the barrel is put back onto the rest of the barrel, we can fit a rubber o-ring in the grooves. These grooves will keep the o-ring in place, and this o-ring will provide an air tight seal when the ball is pushed against it. This will be explained later on, as it is fairly important.

Theory: The physics being taught here break into two categories, that of the launch, and the physics of air resistance. Air resistance will be present in both the launching and flight of the ball, so it will be briefly covered first, followed by an explanation of the launch, and then a more thorough coverage of air resistance.

The projectile motion known by the students is likely the basic component based, neglecting air resistance, projectile motion. It is possible that they do not know the component based part, in which case a quick overview of that will need to be done. I will be asking the teacher closer to the date of the presentation whether or not they know this aspect (I have talked to him already, and says he plans to have it in by then). This type of projectile motion involves only one force, the force due to gravity (F_g). This force will only be applied in the -y direction. The drag force (F_D) will be the force from air resistance and it will almost always be in multiple component, and will always be opposite the direction of travel. The formula looks like

$$F_D(v) = \tfrac{1}{2}\, p\, v^2\, C_D\, A$$

where:
- p is the mass density of the fluid. 1.204 kg/m3 in the case of room temperature air
- v is the velocity of the object
- C_D is the coefficient of drag of the object (for spheres it is .48)
- A is the cross sectional area of the object in the direction of a travel (in this case, it will always be the area of a circle with the diameter of the ball)

The standard diameter of a ping pong ball is 40mm, this gives us

$$F_D(v) = .000363\, v^2$$

This may seem so small it is negligible, but remember the mass of a ping pong ball is 2.7g, or .0027 kg. How we apply this force is by taking the component velocities of the object, and plugging in the velocity, we will find the force being applied. However, as the velocity of the object changes, so too will the force, this will require us to be constantly recalculating the force. I will let the students do this in decently large steps to save time, but will also write a program that will do these calculations at different step sizes. In the end, when the ball is in flight, we will have the x and y forces described as

$$F_x = F_D(v_x)$$
$$F_y = F_D(v_y) + F_g$$

making sure to be careful the the drag forces are always opposite the direction of travel. The x component will never flip signs, however the y component will. From here we use

$$F = m\,a$$

to get the acceleration. now using that acceleration, along with whatever time step size is chosen (t) and the velocities (at this point we call them the initial velocities, or v_i) we can then calculate the displacement of the ball during the time step as well as the velocities at the end of the time step

$$d = v_i + \tfrac{1}{2}\,a\,t^2$$
$$v_f = v_i\,t$$

With these new final velocities, we can then move to the next time step, and use the final velocities as the new initial velocities. We will continue this until we hit the ground (0 in the y direction), and then at that point we will see where we are in the x direction. This of course will be more accurate for smaller time steps, and while the students will just be doing one time step, I will have a computer program able to do the calculations much more accurately (though using the exact same math) ready.

Next we will look at the physics of how the ball is shot. For this part, I have had to simplify the physics a great deal. This is obviously due to the "real" physics being far too complex to cover in a class. These complexities arise from the imperfect aspects of the cannon, from the non instant opening of the release valve, to the fact that the cannon barrel will not be completely snug with the ball. The thermodynamic physics involved in the expansion of a gas also adds another layer of complexity. So in the end, I will be using a very simple model, of an are with constant pressure against it.

$$F_P = P\,A$$

P here is the pressure, and A is the area it is being applied against. We can simply use the cross sectional area of the ball, which will again be a circle of diameter 40mm. The pressure will be assumed to be constant.
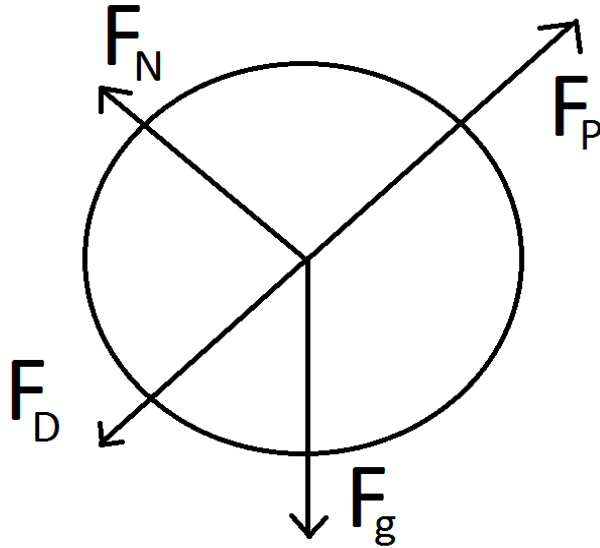
If we look at a free body diagram of the ball (forces are not drawn to scale with each other), we see that there will be several other forces involved when we launch the ball. There will be a drag force FD, the force due to gravity Fg, and the normal force FN. However, due to the large nature of FP relative to all the other forces, they will at most be 5% of that of FP. Also, considering the simplifications already taken, I have decided to use this simplification as well, however I will still show these other forces, and explain this simplification.

An important thing to note here is that initially I will have the students calculate the speed of the ball as if the entire barrel was used. This will not be the case however, as only the last centimeter or so will be used (as the ball gets pushed through the o-ring). The math is the same however, and I will explain to them why their result does not match what we observe.

Demonstration: The demonstration will begin with an introduction to the drag force. This force will be apparent throughout the experiment (even though we will ignore it in the launching), so it will be explained first. I will show the students the formula I will be using for drag

$$F_D(v) = \tfrac{1}{2}\, p\, v^2\, C_D\, A$$

and I will go through the terms, one at a time, explain how each would make sense to have the impact it has. For example, for A, a larger area would mean more drag force; or for CD, a less efficient shape would cause more drag. I will then explain how the force is the result of Newtons third law, it is the opposite and equal force of pushing the air molecules out of the way of the ball. Finally, I can call to their attention that there is no mass in this equation, which means that the drag force through air is entirely dependent on the shape, size and speed of an object, but not its mass.

I can lastly fill in the values of each of the known variables, leaving us with

$$F_D(v) = .000363\, v^2$$

With the basic concept of how the drag force work, I will begin to explain the launch physics. The main

force behind the launch will be due to pressure. Give them the formula

$$F_P = P\,A$$

and begin to explain how it will work. I think that explaining how we can simplify the equation to using just the cross sectional area, when in fact it uses the half sphere may not work well if they do not have a strong grasp of multi-component physics. None the less, I can briefly explain how the area is actually the surface area of the sphere, and the non-parallel to launch forces will cancel out with each other.

With this pressure force in mind I will draw a circle, and have them try to guess the forces involved. This should end up looking like the previous free body diagram.
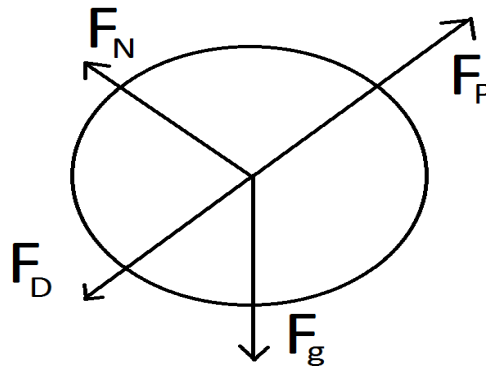
So we now have a net launching force that involves 4 different forces. I can quickly show how the normal force and the perpendicular component of the force due to gravity will cancel out, but that will still leave us with 3 forces. As we will be launching this at a low angle (my current idea is about 30 degrees), I can show how the gravity force will be quite small. I will then have them quickly (or what I hope will be quickly) find what the exit velocity of the ball will be if we assume a constant pressure of ~25 of a psi, or 172 368 pascals. This should lead to them seeing 217N of force, which will be an acceleration of ~80 000 m/s2. Using formulas they should already know

$$d = v_i + \tfrac{1}{2}\,a\,t^2$$
$$v_f = v_i\,t$$

and given the length of the barrel at 25cm, they should see that he ball will leave the barrel at 160m/s. This should not seem like a reasonable speed (Canada defines a gun as something producing a bullet with speed above 152 m/s, and this is not a gun). I will then use a link to a youtube video of the ball being launched in slow motion. We can use this video to determine roughly what the speed is, and it is around 16 m/s. I will then explain why there is such a big difference in the calculated and actual velocities. This is mostly due to the fact that the ball will only be accelerating for around 1cm (as it passes through the o-ring), instead of 25cm.

I can then have them see what the drag force would be on the ball at this top speed. They should come out with a force that results in an acceleration of ~17m/s2, which is negligible compared to the 80 000m/s2 of the pressure force. So now we see why we can ignore everything but the pressure force when dealing with the launch.

So we now have an idea of the launch speed, now we move into guessing the projectile part. We can get 1 more bit of practice with the drag force here that will tie into projectile motion, and that is finding

the terminal velocity of a ping pong ball. This will be fairly simple, as we just want to find the point where the drag force equals the force due to gravity. The equations we have should give us an answer of 8.54m/s, which is within about 10% of the actual measured velocity of 9.5 m/s. So we see that this is a good approximation of the drag force, and we get some more practice using it.

Next, do another free body diagram
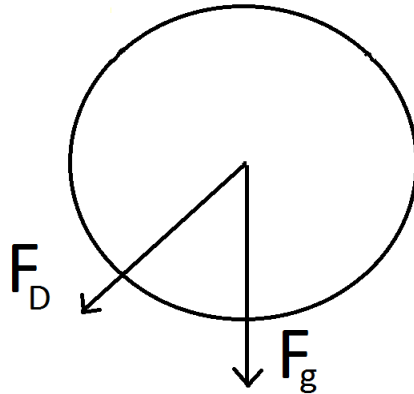
We now have to definitely deal with multiple components, so to make it as easy as possible, show how we can consider the 2 components to each have a velocity and a related force:

$$F_{Net\ x} = F_D(v_x)$$
$$F_{Net\ y} = F_D(v_y) + F_g$$

At this point it is important to again remind them that the drag force is always opposite the direction of travel, and that the drag force is dependent on the current velocity of the object, so it will have to be constantly rechecked. Now I can have them try to do the math for a single step, using a time step size of .1s. With that done, I can then explain how a computer does basically the same thing to calculate the flight of something with air resistance. It will use the same formulas to do the step by step math of the ball in flight. The only difference is that it can do the math a lot quicker, and because of that we can choose much smaller time steps, which result in much smaller errors. I will then have multiple graphs showing how different step sizes will give different predictions, and using these graphs I will have them guess where the ball will land after we launch it. Finally, we can launch the cannon, and see how accurate we were.

Unfortunately, ping pong ball launchers are not amazingly accurate. Some reasons why they won't hit exactly where guessed are that the spin of the ball and the imperfections will greatly affect the trajectory and speed of the ball, and the ball also has small differences from a true sphere. These can add up (the imperfections especially) to a result that is difficult to accurately predict.