Visualizing Hydrostatic Pressure - Demonstration

Once the setup has been completed, the demonstration will begin by removing all of the thumb tacks from each container.

According to Bernoulli's Principle, the value of the sum $\frac{v^2}{2} + gh + \frac{p}{\rho_{water}}$, where v is the speed at which water is flowing, g is the magnitude of the acceleration due to gravity, h is the height, p is pressure, and ρ_{water} is water density, is constant at all points in a system of flowing water.

Once the thumb tacks have been removed from each container shown in Figure 3, water will begin flowing out of the unplugged holes in the container's wall as a result of water pressure.

Because the volume of water flowing out of a pinhole-sized opening created by a thumb tack every second is extremely small compared to the total volume of water inside the container, the speed at which water is flowing at the water surface can be considered to be zero. In addition, since the container's cap has been removed, the pressure at the water surface is equivalent to atmospheric pressure, which is approximately 101.3 kPa at sea level. As a result, $\frac{{v_{water\ surface}}^2}{2} + gh + \frac{p_{water\ surface}}{\rho_{water}} = gh + \frac{101.3kPa}{\rho_{water}}$ at the surface of the water of height h inside the container.

At an opening in the container's wall, the water is in direct contact with the atmosphere. Therefore, the pressure at this point is also equivalent to atmospheric pressure. As a result, $\frac{{v_{wall\ opening}}^2}{2} + g(h-d) + \frac{p_{wall\ opening}}{\rho_{water}} = \frac{{v_{wall\ opening}}^2}{2} + g(h-d) + \frac{p_{wall\ opening}}{\rho_{water}}$, for an opening in the wall at a vertical distance d from the water surface of height h.

According to Bernoulli's Principle,

$$gh + \frac{101.3kPa}{\rho_{water}} = \frac{{v_{wall\ opening}}^2}{2} + g(h-d) + \frac{101.3kPa}{\rho_{water}},$$

$$gd = \frac{{v_{wall\ opening}}^2}{2},$$

$$v_{wall\ opening} = \sqrt{2gd} = \sqrt{19.6\frac{m}{s^2}(d)}.$$

It is clear from the above equation that the speed at which water flows out of an opening in the container wall is only dependent on its distance from the water's surface. Because both of the containers shown in Figure 3 has been filled with water up to the same height prior to having the thumb tacks in their walls removed, water will flow at the same speed out of openings of the same height in both container walls, regardless of the differences in shape and size of the containers. As a result, the water flowing out of openings of the same height will trace out paths with identical curvature, providing an intuitive visual representation of the physical principle that the pressure along a horizontal plane parallel to the surface of the water is constant and that the magnitude of this pressure is dependent on the plane's vertical distance from the surface.

For example, by filling each container with water to a height of 20cm, water will flow out of openings:

- at a height of 18cm with a speed of $\sqrt{19.6\frac{m}{s^2}(0.2m - 0.18m)} \approx 0.636\frac{m}{s}$,
- at a height of 12cm with a speed of $\sqrt{19.6\frac{m}{s^2}(0.2m - 0.12m)} \approx 1.252\frac{m}{s}$, and
- at a height of 2cm with a speed of $\sqrt{19.6\frac{m}{s^2}(0.2m - 0.02m)} \approx 1.878\frac{m}{s}$.

For openings at a height of 18cm, the path traced out by outflowing water can be described by the following parametric equations:

$$x(t) = 0.646\frac{m}{s}(t) \text{ and } y(t) = 0.18m - \frac{1}{2}gt^2 = 0.18m - 4.9\frac{m}{s^2}(t^2),$$

where x is the horizontal distance from the opening and y is the vertical distance from the bottom of the container.

Similarly, the path traced out by outflowing water from openings at a height of 12 cm can be described by the equations:

$$x(t) = 1.252\frac{m}{s}(t) \text{ and } y(t) = 0.12m - 4.9\frac{m}{s^2}(t^2),$$

while the paths traced out by outflowing water from openings at a height of 2cm can be described by:

$$x(t) = 1.878\frac{m}{s}(t) \text{ and } y(t) = 0.02m - 4.9\frac{m}{s^2}(t^2).$$

These paths are shown below in Figure 1.

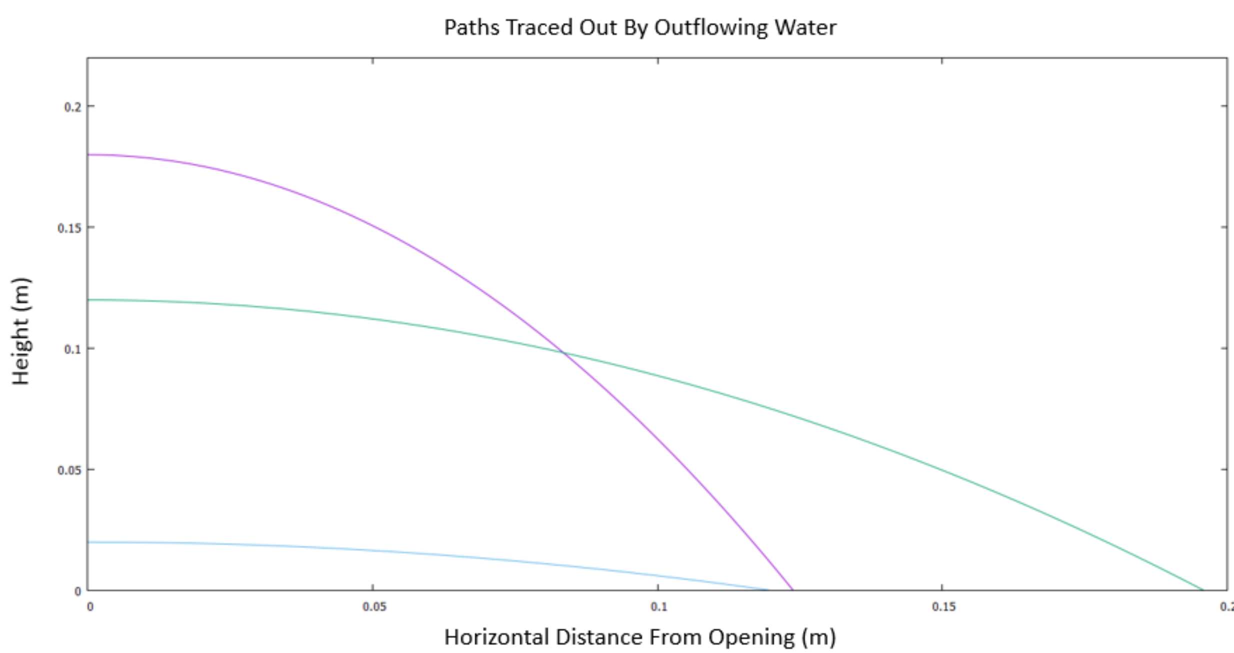

Figure 1 - Paths Traced Out By Outflowing Water